Ultraviolet Completion of Electroweak Theory on Minimal Fractal Manifolds

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Abstract

The experimental discovery of the Higgs boson at the Large Hadron Collider (LHC) has effectively disqualified all Higgs-less models developed prior to July 2012. Today, despite its conclusive validation, the Higgs sector of the Standard Model (SM) remains a largely uncharted territory. This raises the following question: Are there any hidden insights brought up by Higgs-less models that can still be beneficial for the on-going research in particle physics? Pursuing this thought, we re-examine here Moffat's scenario based on a finite electroweak Lagrangian built outside the Higgs paradigm. Unlike the original proposal, we place the model on a spacetime support equipped with minimal fractality. In doing so, we find that the theory is perturbatively well-behaved at large scattering cross-sections and that it gracefully connects with the conventional formulation of the SM in the limit of vanishing fractality.

1. Introduction

It is widely recognized that the properties of the Higgs boson discovered at CERN in 2012 are consistent with the predictions of the SM. However, it is currently unknown if the Higgs boson is the scalar predicted by the SM or some low-energy manifestation of new physics conjectured to come into play beyond SM. There are currently many unsettled questions surrounding the phenomenology of the Higgs sector (see e.g. [1]) and the hope is that the restart of the LHC in 2015 will stimulate further progress on these issues. The present state of affairs suggests that there are valuable insights in Higgs-less approaches which may be relevant for model-building efforts in particle theory. Following up on this thought, we re-visit here Moffat's scenario in which a Higgs-less electroweak (EW) model is rendered finite in the ultraviolet sector (UV) by generalizing the standard scale dependence of gauge couplings [2]. The idea can be expanded by recalling that, in general, the standard running of couplings is equivalent to their evolution in *continuous spacetime dimensions* [3-4]. Drawing on previous works on this topic [3-7, 10-14], we place Moffat's model on a spacetime support equipped with minimal fractality. In doing so, we find that the theory is perturbatively well-behaved at large scattering cross-sections and that it gracefully connects with the conventional formulation of the SM in the limit of vanishing fractality.

Our paper is organized as follows: next section briefly surveys the main points behind Moffat's theory; the relevance of "*continuous dimensions*" and "*minimal fractal manifold*" (MFM) in field theory form the topic of section 3. Next section connects Moffat's theory with the power-law scaling of gauge couplings on the MFM. Section 5 explores the idea that the Higgs scalar discovered at CERN is actually a *Bose-Einstein condensate of gauge bosons* on the MFM. Concluding remarks are presented in the last section.

2. Moffat's electroweak model

Consider a Higgs-less EW model based on the standard local symmetry group $SU_c(3)\times SU_L(2)\times U_Y(1)$ [2]. Assuming natural units ($\hbar = c = 1$) and an underlying metric with signature $\eta_{\mu\nu} = diag(+1,-1,-1,-1)$, the Lagrangian of this model can be broken up into a couple of terms

$$L_{EW} = L_1 + L_2 \tag{1}$$

The first term includes all covariant differential operators,

$$L_1 = \sum_{\psi_L} \overline{\psi}_L [\gamma^\mu (i\partial_\mu - \overline{g}\, T^a W^a_\mu - \overline{g}\,' \frac{Y}{2} B_\mu)]\psi_L + \sum_{\psi_R} \overline{\psi}_R [\gamma^\mu (i\partial_\mu - \overline{g}\,' \frac{Y}{2} B_\mu)]\psi_R - \frac{1}{4}(B^{\mu\nu}B_{\mu\nu} + W^a_{\mu\nu} W^{a\mu\nu})$$

whereas the second term contains the mass contributions for both vector bosons and fermions, namely,

$$L_2 = \frac{M^2}{2}(B^\mu B_\mu + W^{a\mu} W^a_\mu) - \sum_{\psi^i_L, \psi^j_R} m^f_{ij} (\overline{\psi}^i_L \psi^j_R + \overline{\psi}^i_R \psi^j_L)$$

To build a viable quantum field theory that is a) perturbatively complete in the UV, b) Poincaré invariant and c) avoids unitarity violation of scattering amplitudes at large energies, the ordinary gauge couplings of the standard EW Lagrangian need to be upgraded to

$$\overline{g} = g\, E(\frac{p^2}{\Lambda_W^2}) \tag{2a}$$

$$\overline{g'} = g' E(\frac{p^2}{\Lambda_W^2}) \tag{2b}$$

Here, Λ_W is a reference scale and the interpolating function $E(t)$ is an entire function on the complex argument $t = -\frac{p^2}{\Lambda_W^2}$ that satisfies the "on-shell" condition in the low-energy regime

$$\lim_{t \to 1} E(t) = 1,\ \ t < 1 \tag{3a}$$

and drops down in the UV as in

$$\lim_{t \to \infty} E(t) = 0 \tag{3b}$$

Consider now the scattering process of longitudinally polarized W bosons

$$W_L^+ + W_L^- \to W_L^+ + W_L^- \tag{4}$$

The matrix elements of scattering amplitude are given by

$$iA_W = ig^2[\frac{\cos\theta + 1}{8M_W^2}s + O(1)] \tag{5}$$

where θ is the scattering angle and $\sqrt{s}$ is the center-of-mass energy. Addition of the SM Higgs boson cancels unitarity violation at large s in (5) due to the Higgs contribution

$$iA_H = -ig^2[\frac{\cos\theta + 1}{8M_W^2}s + O(1)] \tag{6}$$

By contrast, unitarity violation in Moffat's model is suppressed by amplitudes vanishing off at large s as a result of (3b), namely,

$$iA_W = -i\,\overline{g}^2(s)[\frac{\cos\theta + 1}{8M_W^2}s + O(1)] \tag{7}$$

$$\lim_{s\to\infty} \overline{g}^2(s) = 0 \tag{8}$$

<u>3. The minimal fractal manifold (MFM)</u>

Two immediate questions arise regarding Moffat's model:

1) *What underlying principle motivates the existence of the interpolating functions introduced in (2a) and (2b)?*

2) *Is the entire model refuted by the discovery of the Higgs boson?*

The next sections attempt to answer these questions. We first show that the interpolating functions of (2a-b) naturally occur if the underlying spacetime is no longer considered a smooth continuum but it is allowed to have arbitrarily small deviations from four dimensions ($D = 4 \pm \varepsilon$, ε << 1). In what follows, we refer to such spacetime as a "*minimal fractal manifold"* (MFM). We indicate that a particular class of models defined on the MFM restores unitarity of scattering amplitudes in UV without invoking the Higgs mechanism. Then we show that this class of models does not exclude the existence of a scalar resonance replicating the attributes of the SM Higgs but consisting of a *condensate of gauge bosons* on the MFM.

As pointed out in [4-6, 10, 14], the onset of fractal geometry in QFT follows from the path toward criticality near the EW scale. It is known that the most reliable description of criticality is through the tools of the Renormalization Group program (RG), in general and dimensional regularization, in particular. Regularization techniques devised in RG are not independent from each other. The connection between dimensional and UV cutoff regularizations stems from the minimal subtraction scheme ($\overline{MS}$) and is given by [5-6]

$$\log \frac{\Lambda_{UV}^2}{\mu^2} = \frac{2}{\varepsilon} - \gamma_E + \log 4\pi + \frac{5}{6} \tag{9}$$

Here, γ_E stands for the Euler constant, μ for the observation scale and Λ_{UV} for the UV cutoff. It is more convenient to present (9) is a slightly different form, that is,

$$\varepsilon \sim \frac{1}{\log(\frac{\Lambda_{UV}^2}{\mu^2})} \tag{10}$$

If the numerical disparity between μ and Λ_{UV} is large enough, one can reasonably approximate ε as in

$$\varepsilon \sim (\frac{\mu}{\Lambda_{UV}})^2 \tag{11}$$

It is apparent from (10) or (11) that the four-dimensional space-time continuum is recovered in either one of these limits:

a) $\Lambda_{UV} \to \infty$ and $0 < \mu << \Lambda_{UV}$,

b) $\Lambda_{UV} < \infty$ and $\mu \to 0$

However, both limits are disfavored by our current understanding of the far UV and the far IR boundaries of field theory. Theory and experimental data alike tell us that the notions of infinite *or* zero energy are, strictly speaking, meaningless. This is to say that either infinite energies (point-like objects) or zero energy (infinite distance scales) are *unphysical idealizations*. Indeed, there is always a finite cutoff at both ends of either energy or energy density scale (far UV = Planck scale, far infrared (IR) = finite radius of the observable Universe or the non-vanishing energy density of the vacuum set by cosmological constant). These observations are also consistent with the estimated infinitesimal (yet non-vanishing) photon mass, as discussed in [11-12].

A key feature of the MFM is that the assumption ε << 1, postulated near the EW scale, is the *only sensible way* of asymptotically matching all consistency requirements mandated by relativistic QFT and the SM [5-6]. In particular, large departures from four-dimensionality imply non-differentiability of spacetime trajectories in the conventional sense. This in turn, spoils the very concept of "speed of light" and it becomes manifestly incompatible with the Poincaré symmetry.

4. Moffat's theory as manifestation of the MFM

To make progress from this point on, we appeal to the recently developed methods of *fractional field theory* [5, 8-9]. Specifically, we assume that field theory defined on fractional four-dimensional spacetime is described by the action

$$S = \int_{-\infty}^{+\infty} d\rho(x) L = \int_{-\infty}^{+\infty} (\mathrm{v}(x)\, d^4x) L \tag{12}$$

where the measure $d\rho(x)$ denotes the ordinary four-dimensional volume element multiplied by a weight function $\mathrm{v}(x)$. If the weight function is factorizable in coordinates and positive semidefinite, $\mathrm{v}(x)$ takes the form

$$\mathrm{v}(x) = \prod_{\eta=0}^{3} \frac{\left|x^{\eta}\right|^{\alpha_{\eta}-1}}{\Gamma(\alpha_{\eta})} \tag{13}$$

in which

$$0 < \alpha_{\eta} \leq 1 \tag{14}$$

are four independent parameters. An isotropic spacetime of dimension $D = 4 \pm \varepsilon$ is characterized by

$$\alpha = 1 \pm \varepsilon = \frac{\sum_{\eta} \alpha_{\eta}}{4} \tag{15}$$

which turns (13) into

$$\mathrm{v}(x) \approx (|x|^4)^{\pm\varepsilon} \tag{16}$$

Dimensional analysis requires all coordinates entering (13) and (16) to be scalar quantities. They can be generically specified relative to a characteristic length and time scale, as in

$$x = \frac{x_0}{L} = \frac{\mu}{\mu_0} \tag{17}$$

in which $\mu,\ \mu_0$ are positive-definite energy scales. Relation (16) becomes

$$\mathrm{v}(x) = (\frac{\mu}{\mu_0})^{\pm 4\varepsilon} \tag{18}$$

such that, in the IR limit $|x| \to 0$ and under the strong assumption that ε is *at least* an order of magnitude larger than $|x|$, e.g. $|\varepsilon| >> |x|$,

$$\lim_{|x|\to 0} \mathrm{v}(x) = \begin{cases} 0, \ if \ \pm\varepsilon > 0 \\ \infty, \ if \ \pm\varepsilon < 0 \end{cases} \tag{19}$$

In the UV limit $|x| \to \infty$ and under the strong assumption $|\varepsilon| >> |x|^{-1}$, (18) behaves in a complementary way to (19), namely,

$$\lim_{|x|\to \infty} \mathrm{v}(x) = \begin{cases} \infty, \ if \ \pm\varepsilon > 0 \\ 0, \ if \ \pm\varepsilon < 0 \end{cases} \tag{20}$$

A remarkable property of the MFM is the emergence of "effective" coupling charges depending on the weight function as in [8-9]

$$\overline{g}^2 = g^2 \mathrm{v}(x) \tag{21a}$$

$$\overline{g'}^2 = g'^2 \mathrm{v}(x) \tag{21b}$$

in which g, g' are charges residing on the four-dimensional spacetime ($D = 4$). Combining (2), (7)-(8) and (21) leads us to conclude that, *at least in principle*, MFM is capable of securing unitarity of scattering amplitudes in Moffat's Higgsless model.

5. Higgs scalar as Bose-Einstein condensate on the MFM

The intent of this section is to show that Moffat's model built on the MFM does not exclude the existence of a scalar resonance replicating the attributes of the SM Higgs but emerging as *condensate of gauge bosons* on the MFM. To some extent, the condensation process bears similarities with Anderson's localization of quantum waves in random media.

It was argued in [13] that the transition from order to chaos in classical and quantum systems of gauge and Higgs fields is prone to occur somewhere in the low to mid TeV scale. The inability of the Higgs vacuum to survive not too far above the LHC scale explains away the fine-tuning problem and signals the breakdown of the SM in this region. The likely instability of the vacuum in the low to intermediate TeV scale brings up an intriguing speculation on the nature of the Higgs scalar. As suggested in [14], scalars are the most likely to form a Higgs-like condensate of gauge bosons on MFM, that is,

$$\Phi_C = \frac{1}{4}[(W^+ + W^- + Z^0 + \gamma + g) + (W^+ + W^- + Z^0 + \gamma + g)] \tag{22}$$

As further explained in [14], a remarkable feature of (22) is that it is a weakly coupled cluster of gauge fields having *zero topological charge*. Compliance with this requirement motivates the duplicate construction of (22), which contains individual *WW*, *ZZ*, photon and gluon doublets. Stated differently, (22) is the only inclusive combination of gauge field doublets that is free from all gauge and topological charges. Table 1 shows a comparative display of properties carried by the SM Higgs and the Higgs-like condensate.

Scalar field	**Original form**	**Composition**	**Mass (GeV)**	**Weak hypercharge**	**Electric charge**	**Color**	**Topological charge**
SM Higgs	$\begin{pmatrix} \varphi^{+} \\ \varphi^{0} \end{pmatrix}$	none	~ 126	$\begin{pmatrix} +1 \\ +1 \end{pmatrix}$	$\begin{pmatrix} +1 \\ 0 \end{pmatrix}$	0	0
Higgs-like condensate	Φ_C	(22)	~ 126	0	0	0	0

Tab. 1: SM Higgs doublet versus the Higgs-like condensate.

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